AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

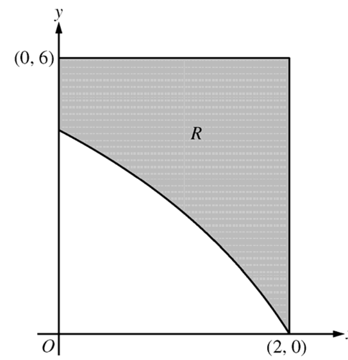
Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.



1 : Correct limits in an integral in (a), (b), or (c)

(a) $\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or 6.817

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right) dx$
$= 168.179$ or 168.180

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^2 (6 - 4\ln(3 - x))^2\,dx = 26.266$ or 26.267

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

1A,

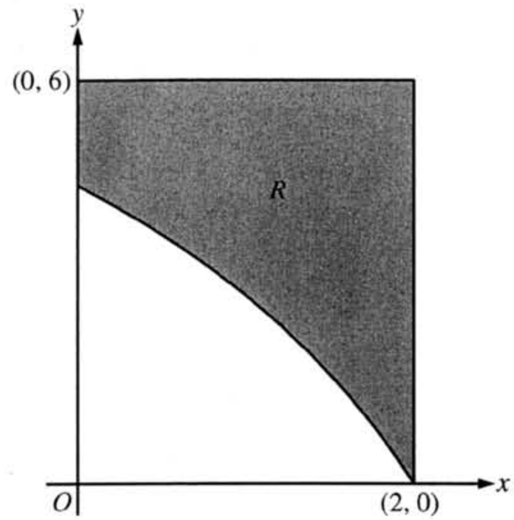
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$SR = 2\times 6 - \int_0^2 4\ln(3-x)\,dx$

$\doteq \cancel{}\, 12 - (5.183) \approx 6.817$

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Continue problem 1 on page

1A₂

Work for problem 1(b)

assum $y_1 = 4\ln(3-x)$

$y_2 = 6$

$$V = \cancel{\pi \int_0^2 \left[(8-y_1)^2 - (8-y_2)^2 \right] dx}$$

$$= \pi \int_0^2 \left[(8-y_1)^2 - (8-y_2)^2 \right] dx$$

$$= \pi \int_0^2 \left[(8-4\ln(3-x))^2 - (8-6)^2 \right] dx$$

$$\approx 168.180.$$

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Work for problem 1(c)

$$V = \int_0^2 \left[6 - 4\ln(3-x) \right]^2 \, dx.$$

$$\approx 26.267.$$

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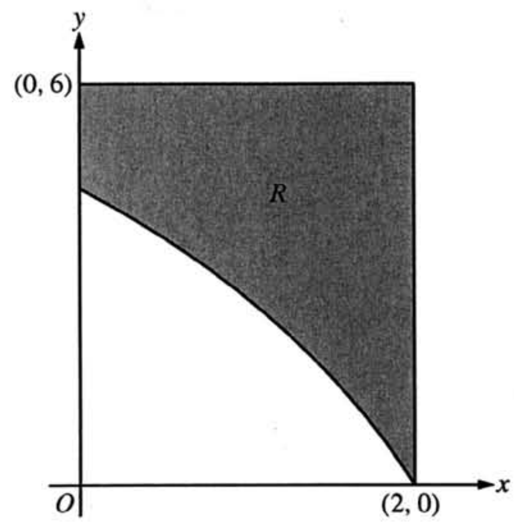
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$R=\int_0^2 6-4\ln(3-x)\,dx$

$=\int_0^2 6\,dx-\int_0^2 4\ln(3-x)\,dx$

$=6x\big|_0^2-4\int_0^2\ln(3-x)\,dx$

$f=\ln(3-x)\quad f'=\frac{-1}{3-x}$

$g'=1\quad g=x$

$=12-\left(4x\ln(3-x)\big|_0^2-\int_0^2\frac{-x}{3-x}\,dx\right)$

$=6.817$ units^2

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Continue problem 1 on page 5.

Work for problem 1(b)

$$V = \pi \int_0^2 [8-4\ln(3-x)]^2 - (8-6)^2 \, dx$$

$$= \pi \int_0^2 \left([8-4\ln(3-x)]^2 - 2^2 \right) dy$$

$$= 168.180 \text{ units}^3$$

Work for problem 1(c)

$$V = \int_0^2 (2\times 6)^2 - (2\times 4\ln(3-x)^2 \, dy$$

$$= \int_0^2 12^2 - (8\ln(3-x))^2 \, dx$$

$$= 222.133 \text{ units}^3$$

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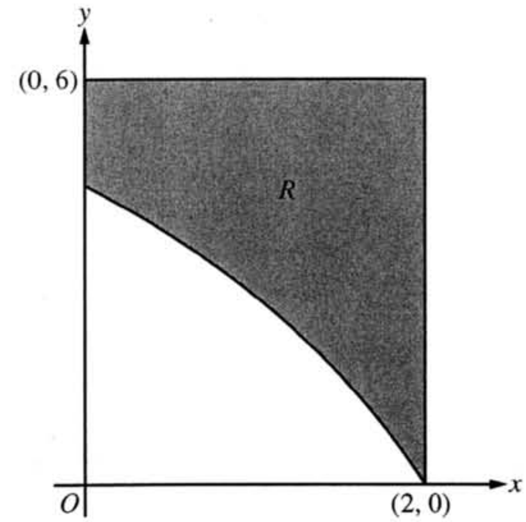
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

$A = \int_0^2 (6 - 4\ln(3-x))\,dx$

with the help of the calculator, we type this in and we get $A = \int_0^2 (6-4\ln(3-x))\,dx = 6.81$

Continue problem 1 on page

Work for problem 1(b)

Since ~~the~~ it R is revolved about $l: y=8$
we use the washer method

$$V=\pi\int_0^2 (8-6^2)(8-(4\ln(3-x))^2)\,dx$$

with the calulator, we get

$$V=\pi\int_0^2 (8-6^2)(8-(4\ln(3-x))^2)\,dx = 41.059$$

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Work for problem 1(c)

The side of the square shall be $y=4\ln(3-x)$
which makes the Area of the square as $A=(4\ln(3-x))^2$

$$V=\pi\int_0^2 (4\ln(3-x))^2\,dx$$

We use the calulator to get

$$V=\pi\int_0^2 (4\ln(3-x))^2\,dx = 51.732$$

GO ON TO THE NEXT PAGE.

AP® CALCULUS BC
2010 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: the global limits point, 2 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student earned both points and the global limits point. The student's intermediate work includes a misplaced 4, but the correct numerical answer was treated as a restart since this was the calculator portion of the exam. In part (b) the student's work is correct. In part (c) the student does not use square cross sections and was not eligible for any points.

Sample: 1C
Score: 3

The student earned 3 points: the global limits point, 2 points in part (a), no points in part (b), and no points in part (c). In part (a) the student earned the global limits point and has correct work. In part (b) the student attempts to find the volume using washers, but the work is incorrect. In part (c) the student uses an incorrect width for the area of the square cross section and includes a factor of π.

AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 2

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0,$ the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5,$ find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the acceleration vector of the particle at $t = 1$.

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \ne 0$.

On $0 < t < 1.5,$ this happens at $t = 1.253$ and $t = 1.144$ or 1.145.

$$2 : \begin{cases} 1 : \text{sets } \dfrac{dx}{dy} = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $$\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)} = 0.863447$$

$$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$$

$$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$$

The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.

$$4 : \begin{cases} 1 : \left.\dfrac{dy}{dx}\right|_{t=1} \\ 1 : x(1) \\ 1 : y(1) \\ 1 : \text{equation} \end{cases}$$

(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$

1 : answer

(d) Acceleration vector: $\langle x''(1), y''(1)\rangle = \langle -28.425, 2.161\rangle$

$$2 : \begin{cases} 1 : x''(1) \\ 1 : y''(1) \end{cases}$$

Work for problem 2(a)

Because the line tangent to the path of the partical is vertical

$\frac{dx}{dt} = 0$

$14\cos(t^2)\sin(e^t) = 0$

for $0 < t < 1.5$ $t = 1.145$ or $t = 1.253$

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Work for problem 2(b)

the slope of the particle's path at $t=1$

is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)} = \frac{1+2\sin(1)}{14\cos(1)\sin(e)}$

$= 0.863$

~~the path vector of the particle is~~

~~$(\int 14\cos(t^2)\sin(e^t), \int 1+2\sin(t^2))$ with a position $(-2,3)$ at $t=0$~~

The position of the particle at $t=1$ is

$(-2 + \int_0^1 14\cos(t^2)\sin(e^t)\,dt, 3 + \int_0^1 1+2\sin(t^2)\,dt)$

$(9.315, 4.621)$

Therefore, the line tangent to the path of the particle at $t=1$

is $y - 4.621 = 0.863(x - 9.315)$

Continue problem 2 on page 7.

Work for problem 2(c)

The speed of the particle at t=1 is

$\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2} = \sqrt{[14\cos(t^2)\sin(e^t)]^2+[1+2\sin(t^2)]^2}$

$= \sqrt{(14\cos 1 \sin e)^2+(1+2\sin 1)^2}$

$= 4.105$

Work for problem 2(d)

The acceleration vector of the particle at $t=1$ is

$(\frac{d}{dt}(\frac{dx}{dt}), \frac{d}{dt}(\frac{dy}{dt}))$

$(14(-\sin(t^2)\cdot 2t\sin(e^t)+\sin(e^t)e^t\cos t^2, 2\cos(t^2)\cdot 2t)$

$\langle -28.435, 2.161\rangle$

 2B1

Work for problem 2(a)

slope of the tangent $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)}$.

When $\frac{dx}{dt} = 0$ the slope $\to \infty$ the line will be vertical

Let $14\cos(t^2)\sin(e^t) = 0$ $0 < t < 1.5$

$t_1 = 1.1447$.
$t_2 = 1.2533$.

Work for problem 2(b)

When $t=1$
$$\text{Slope} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{1+2\sin 1}{14\cos 1 \sin e} = a$$

$a = 0.8634$.

$x = \int \frac{dx}{dt} = \int 14\cos(t^2)\sin(e^t)\, dt$.
$y = \int \frac{dy}{dt} = \int 1+2\sin(t^2)\, dt = t - \frac{2\cos t^2}{t} + C$.

$y(t=0) = 0 - 0 + C = 3$ $C = 3$.

Continue problem 2 on page 7.

Work for problem 2(c)

when $t=1$

$\frac{dx}{dt} = v(x) = 14\cos 1 \cdot \sin e = a \quad a = 3.1072$

$\frac{dy}{dt} = v(y) = 1 + 2\sin 1 = b \quad b = 2.6829$

speed $= \sqrt{v(x)^2 + v(y)^2} = \sqrt{a^2+b^2} = c$

$c = \text{speed} = 4.1052$

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Work for problem 2(d)

acceleration $(x) = \frac{d^2x}{dt^2} = (14\cos(t^2)\,\cancel{\sin e}\ t)'$

$= 14\cos(t^2)\cos(e^t)\,e^t - 14\sin(t^2)\,2t \cdot \sin e^t$

acceleration $(x, t=1) = 14\cos 1 \cos(e) \cdot e - 14\sin(1)\,2 \cdot \sin e = d$

$d = -28.4253$

acceleration $(y) = \frac{dy^2}{dt^2} = (1+2\sin(t^2))'$

$= 0 + 2\cos(t^2)\,2t$

acceleration $(y, t=1) = 0 + 2\cos(1) \cdot 2 = f$

$f = 2.1612$

Thus the acceleration vector at $t=1$ is $\langle -28.4253, 2.1612 \rangle$

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Work for problem 2(a)

solve $\frac{dx}{dt}=0$, yields

$\therefore \cos t^2=0$ or $\sin(e^x)=0$.

$\therefore t=\frac{\sqrt{2}}{2}\sqrt{\pi}$ or $t=\ln\pi$

$\therefore$ when ~~t~~ $t=1.253$ or $t=1.145$, the line tangent to the path of the particle is vertical.

Work for problem 2(b)

$$\frac{dy}{dx}=\frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)}\Big|_{t=1}=\frac{1+2\sin 1}{14\cos 1\cdot\sin e}\doteq 0.209$$

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Continue problem 2 on page 7.

Work for problem 2(c)

$$V(1) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{[14\cos(t^2)\sin(e^t)]^2 + [1+2\sin(t^2)]^2}\ \Big|_{t=1}.$$

$\doteq 4.105.$

Work for problem 2(d)

$$a(1) = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = 28.5073$$

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points. In part (a) an ideal solution would include that $\frac{dy}{dt} \neq 0$ at the two points. In part (d) the student's intermediate symbolic work contains an error. Since this question was on the calculator portion of the exam, it was presumed that the student corrected the error when producing a correct numerical result with the calculator.

Sample: 2B
Score: 6

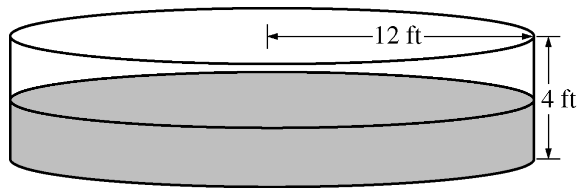
The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student correctly evaluates $\frac{dy}{dx}$ at $t = 1.$ In parts (c) and (d), the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student's numerical slope value is incorrect. In part (c) the student's work is correct. In part (d) the student's work is incorrect.

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

1 : answer

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi (12)^2 h$

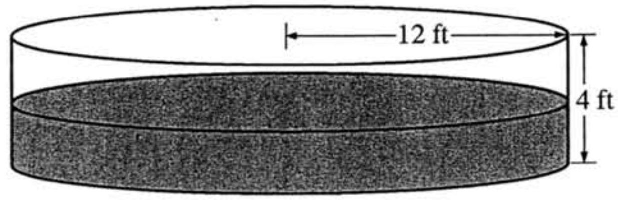
$\dfrac{dV}{dt} = 144\pi \dfrac{dh}{dt}$

$\left.\dfrac{dh}{dt}\right|_{t=8} = \dfrac{1}{144\pi} \cdot \left.\dfrac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{ equation relating } \dfrac{dV}{dt} \text{ and } \dfrac{dh}{dt} \\ 1 : \left.\dfrac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

3A1

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

WATER ADDED INTO POOL $= \int_0^{12} P(t)\,dt \approx 4(46+57+62) = \boxed{660 \text{ ft}^3}$

ABOUT $\underline{660}$ ft^3 OF WATER ARE ADDED TO THE POOL FROM $t = 0\text{h}$ TO $t = 12\text{ h}$

Work for problem 3(b)

WATER LEAKED $= \int_0^{12} R(t)\,dt = \int_0^{12} (25e^{-.05t})\,dt = \boxed{225.594 \text{ ft}^3}$

$\underline{225.594}$ ft^3 OF WATER LEAK FROM THE POOL FROM $t = 0\text{h}$ TO $t = 12\text{h}$

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Continue problem 3 on page 9.

Work for problem 3(c)

$V = \left(\begin{smallmatrix}\text{INITIAL}\\ \text{WATER}\end{smallmatrix}\right) + (\text{WATER IN}) - (\text{WATER OUT}) = 1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1660 - 225.594 =$

$\boxed{= 1434.406\ ft^3}$

$\Downarrow$

$\boxed{1434\ ft^3}$

THE VOLUME OF WATER IN THE POOL AT TIME $t = 12\ h$ IS ABOUT 1434 ft^3

Work for problem 3(d)

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$43.242 = 452.389 \frac{dh}{dt}$

$\frac{dh}{dt} = .096\ ft/hour$

$\frac{dV}{dt} = (\text{RATE WATER IN}) - (\text{RATE WATER OUT})$

$\frac{dV}{dt} = P(8) - R(8) = 43.242\ ft^3/hour$

AT $t = 8\ h$, THE VOLUME IN THE TANK IS INCREASING AT 43.242 $ft^3/hour$

AT $t = 8\ h$, THE WATER LEVEL IS RISING AT .096 ft/hour

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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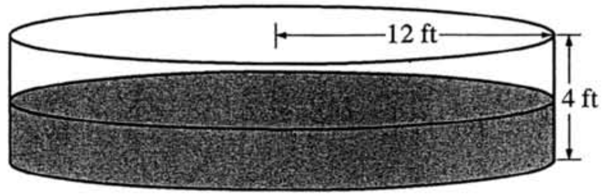
3B1

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



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Work for problem 3(a)

Midpoints are $t=2, 6, 10$, $p(t)=46, 57, 62$.

Sum $= 4(46+57+62) = 660$.

Work for problem 3(b)

$V_{\text{water leaking out}} = \int_0^{12} R(t)\,dt = \int_0^{12} 25e^{-0.05t}\,dt$.

$= 225.594$

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Continue problem 3 on page 9.

3B2

Work for problem 3(c)

$V = V_{pumped} - V_{leak}$
$= 660 - 225.6 = 434.4$ cubic feet.
≈ 434 cubic feet.

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Work for problem 3(d)

$\frac{d}{dt}[p(t) - R(t)] = \frac{d}{dt}(80 - 25e^{-0.05t})$

$= -25 \cdot (-0.05t) \cdot e^{-0.05t-1}$

since $t = 8 \to +25 \times 0.05 \times 8 \times e^{-0.05\times 8-1}$.
$= 2.467$.

Thus, the volume of water is increasing at the rate of 2.467 ft^3/h at $t=8$

~~$V = \pi r^2 h = 2.467$~~. $V = \pi r^2 h$ $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$.

$\frac{dh}{dt} = \frac{2.467}{\pi \cdot 12^2} = 0.0055 ft$.

The water level is rising at the rate of 0.0055 ft/h at $t=8$.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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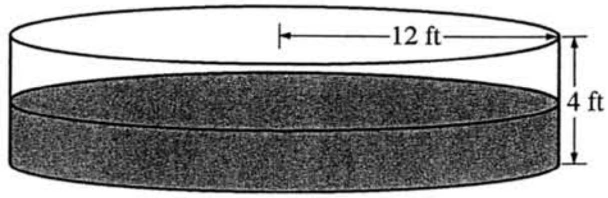
3C1

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

The approximate total amount of water

$$= \frac{(0+53)\times 4}{2} + \frac{(53+60)\times 4}{2} + \frac{(60+63)\times 4}{2}$$

$$= 289 \times 2$$

$$= 578 \text{ cubic feet}$$

Work for problem 3(b)

The total amount of water leaking out

$$= \int_0^{12} 25e^{-0.05t}\,dt = 255.594 \text{ cubic feet}$$

Continue problem 3 on page 9.

3C2

Work for problem 3(c)

The approximate volume of water in time $t=12$

$= 1000 + 578 - 255.594$

$= 1322$ cubic feet

Work for problem 3(d)

The rate $= \frac{d}{dt}(60 - 25e^{-0.05t}) = 1.25 \cdot e^{-0.05t}$

$= 0.8379$ cubic feet per second

The rate of rising $= \frac{d}{dt}(1.25 \cdot e^{-0.05t})$

$= -0.0625 \cdot e^{-0.05t}$

$= -0.419$ cubic feet per square second

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

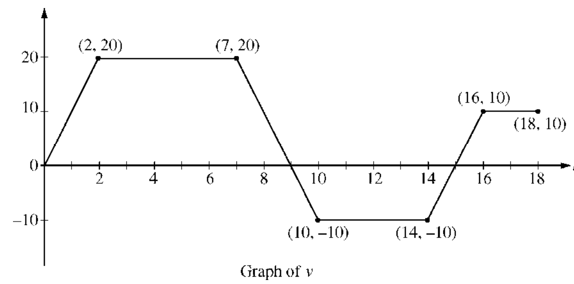
The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student does not use the initial condition, and the point was not earned. In part (d) the student's presented value for $V'(8)$ is incorrect. The relationship between $\frac{dV}{dt}$ and $\frac{dh}{dt}$ is correct, and the value of $\frac{dh}{dt}$ is consistent with the student's $V'(8)$. The second and third points were earned. The units on $\frac{dh}{dt}$ are incorrect.

Sample: 3C
Score: 3

The student earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not use a midpoint Riemann sum. In part (b) the student's work is correct. In part (c) the student correctly combines the results from parts (a) and (b) along with the initial condition. In part (d) the student's work is incorrect.

AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 4



A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2 : \begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2 : \begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\, dt = 140 + 50 + 25 = 215.$

1 : answer

(d) For $7 < t < 10$, $a(t) = \dfrac{20 - (-10)}{7 - 10} = -10$

$$v(t) = 20 - 10(t - 7) = -10t + 90$$

$$x(7) = \frac{7+5}{2} \cdot 20 = 120$$

$$\begin{aligned} x(t) &= x(7) + \int_7^t (-10u + 90)\, du \\ &= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t} \\ &= -5t^2 + 90t - 265 \end{aligned}$$

$4 : \begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$

NO CALCULATOR ALLOWED

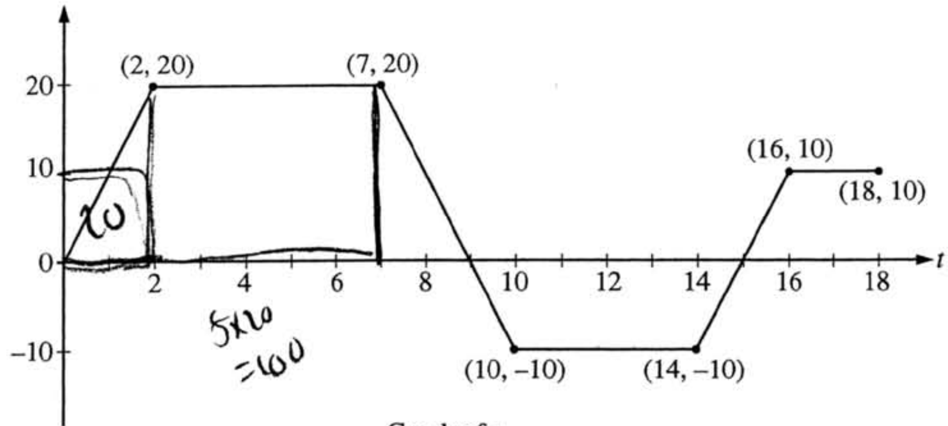
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

The squirrel changes direction for $t=15$ and $t=9$ because velocity changes from negative to positive and vice versa on those points.

Work for problem 4(b)

distance of squirrel from A, at t : $S(t)$

$S(9) = \int_0^9 v(t)\,dt = 140$

$S(15) = \int_0^{15} v(t)\,dt = 140 - 50 = 90$

$S(18) = \int_0^{18} v(t)\,dt = 90 + 25 = 115$

$\therefore$ The squirrel is farthest from the building when $t=9$. The squirrel is 140 away from the building A

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Continue problem 4 on page 1

Work for problem 4(c)

$\int_0^{18} |v(t)| = 140 + 50 + 25 = \underline{215}$

Work for problem 4(d)

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in $(7,10)$

$a(t) = v'(t) = \frac{-10-20}{10-7} = \frac{-30}{3} = \underline{-10}$

$v(9) = 0$

velocity: $y - 0 = -10(x-9)$

$y = -10x + 90$

$\therefore \underline{v(t) = -10x + 90}$

$x(t) = x(7) + \int_7^t v(t)\,dt$

$= 120 + [-5x^2 + 90x]_7^t$

$= 120 + (-5t^2 + 90t) - (385)$

$= \underline{-5t^2 + 90t - 265}$

NO CALCULATOR ALLOWED

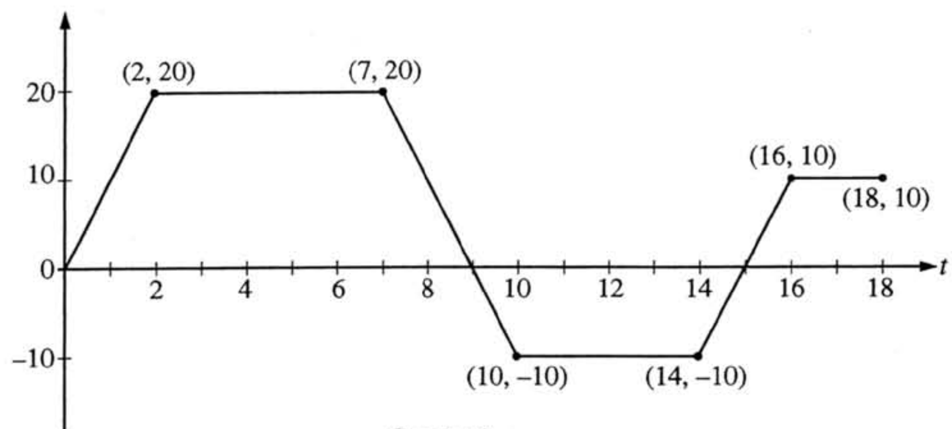
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

The squirrel changes direction at $t=9$ and $t=15$. His velocity changes from positive to negative.

Work for problem 4(b)

At $t=9$ the squirrel is farthest from the building A. At $t=9$, the squirrel is 140 units away from building A.

$\frac{1}{2}\cdot 20\cdot(9+5)=140$

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Continue problem 4 on page 1

NO CALCULATOR ALLOWED

Work for problem 4(c)

$\frac{1}{2}\cdot 20\cdot(14)+\frac{1}{2}\cdot 10\cdot(2+3)+\frac{1}{2}\cdot 10\cdot(6+4)$

$140 + 25 + 50 = 215$

Total distance traveled = 215 units.

Work for problem 4(d)

$\boxed{a(t) = -10}$

$\frac{-10-20}{10-7} = \frac{-30}{3} = -10$

$\boxed{v(t) = -10x + 90}$

$x(t) = -5x^2 + 90x + 120$

$C = \frac{1}{2}\cdot 20\cdot(5+7)$ $\int -10x + 90\, dx$

$C = 120$ $-5x^2 + 90x + C$

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NO CALCULATOR ALLOWED

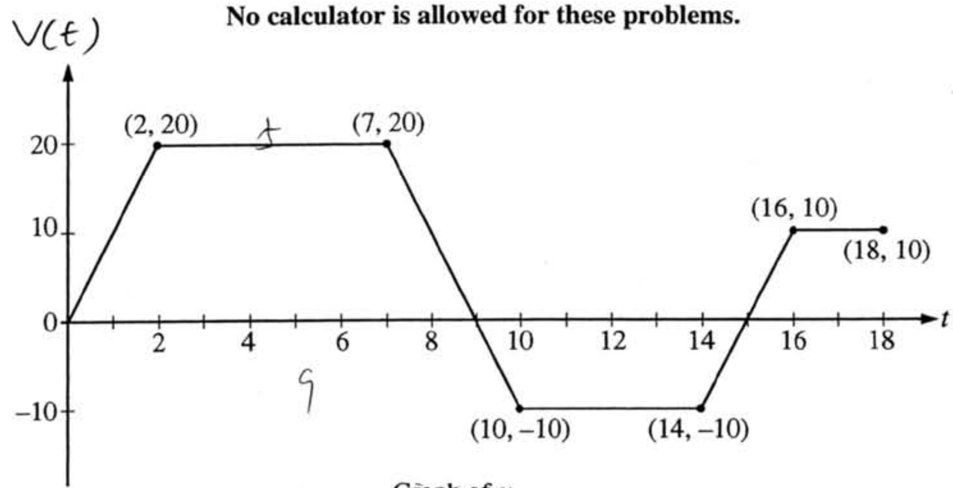
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

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Work for problem 4(a)

1. at $9 < t < 15$, the squirrel changes its direction since its velocity changes from positive to negative.

Work for problem 4(b)

(1) at $t = 9$. because that's when the area between the graph of $v(t)$ and the x-axis is the largest.

(2) $S = \dfrac{(5+9)\times 20}{2} = 140$

Continue problem 4 on page

Work for problem 4(c)

Total Distance :

$= \left| \int_0^9 v(t)\,dt \right| - \left| \int_9^{15} v(t)\,dt \right| + \left| \int_{15}^{18} v(t)\,dt \right|$

$= 140 - 50 + 25$

$= 115$

Work for problem 4(d)

$v(t)$ @, $7<t<10$ is a straight line.

passing $(7,20), (10,-10)$

$\therefore\ v(t) = \underline{-10t+90}.$

According to Motion Theroms,

$a(t) = v'(t) = \underline{-10}$

$x(t) = \int v(t)\,dt = \underline{-5t^2+90t}$ ...

GO ON TO THE NEXT PAGE

Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student identifies the two points at which the graph of v crosses the t-axis but does not correctly explain why the squirrel changes direction at those two points. The given explanation applies to only one of the two points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9.$ The second point was earned. In part (c) the student's work is correct. In part (d) the student has correct expressions for $a(t)$ and $v(t)$, but the expression for $x(t)$ does not incorporate the initial condition. One of the points for $x(t)$ was earned.

Sample: 4C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d). In part (a) the student presents an interval instead of points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9.$ The second point was earned. In part (c) the student finds displacement rather than total distance traveled. In part (d) the student has correct expressions for $a(t)$ and $v(t)$ but not for $x(t)$.

Question 5

Let f and g be the functions defined by $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{4x}{1 + 4x^2}$, for all $x > 0$.

(a) Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0, \infty)$ if the minimum exists. Justify your answers.

(b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x = 1$, below the graph of f, and above the graph of g.

(a) $$g'(x) = \frac{4\left(1 + 4x^2\right) - 4x(8x)}{\left(1 + 4x^2\right)^2} = \frac{4\left(1 - 4x^2\right)}{\left(1 + 4x^2\right)^2}$$

For $x > 0$, $g'(x) = 0$ for $x = \dfrac{1}{2}$.

$g'(x) > 0$ for $0 < x < \dfrac{1}{2}$

$g'(x) < 0$ for $x > \dfrac{1}{2}$

$g\left(\dfrac{1}{2}\right) = 1$

Therefore g has a maximum value of 1 at $x = \dfrac{1}{2}$, and g has no minimum value on the open interval $(0, \infty)$.

$$5 : \begin{cases} 2 : g'(x) \\ 1 : \text{critical point} \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$$

(b) $$\int_1^{\infty} (f(x) - g(x))\, dx = \lim_{b \to \infty} \int_1^b (f(x) - g(x))\, dx$$

$$= \lim_{b \to \infty} \left(\ln(x) - \frac{1}{2}\ln\left(1 + 4x^2\right) \right) \Bigg|_{x=1}^{x=b}$$

$$= \lim_{b \to \infty} \left(\ln(b) - \frac{1}{2}\ln\left(1 + 4b^2\right) + \frac{1}{2}\ln(5) \right)$$

$$= \lim_{b \to \infty} \ln\left(\frac{b\sqrt{5}}{\sqrt{1 + 4b^2}} \right)$$

$$= \lim_{b \to \infty} \ln\left(\frac{\sqrt{5b^2}}{\sqrt{1 + 4b^2}} \right)$$

$$= \frac{1}{2} \lim_{b \to \infty} \ln\left(\frac{5b^2}{1 + 4b^2} \right)$$

$$= \frac{1}{2}\ln\frac{5}{4}$$

$$4 : \begin{cases} 1 : \text{integral} \\ 2 : \text{antidifferentiation} \\ 1 : \text{answer} \end{cases}$$

NO CALCULATOR ALLOWED

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Work for problem 5(a)

$$g(x) = \frac{4x}{1+4x^2}$$

$$g'(x) = \frac{4\cdot(4x^2+1) - 4x\cdot 8x}{(1+4x^2)^2}$$

$$= \frac{-16x^2+4}{(1+4x^2)^2} = -16x\,\frac{(x^2-\frac{1}{4})}{(1+4x^2)^2} = -16\,\frac{(x-\frac{1}{2})(x+\frac{1}{2})}{(1+4x^3)^2}$$

when $0<x<\frac{1}{2} \Rightarrow g'(x) > 0$.

when $x > \frac{1}{2} \Rightarrow g'(x) < 0$.

if $x \to \infty \Rightarrow g(x)$ converges to 0.

So there is no minimum value

but there is the maximum when $x = \frac{1}{2}$

$\hookrightarrow g(\frac{1}{2}) = \frac{2}{1+1} = 1$.

answer: no minimum

maximum $= 1$

(when $x = \frac{1}{2}$)

Continue problem 5 on page 13.

Work for problem 5(b)

$$\text{area} = \int_1^\infty |g(x) - f(x)|\, dx = \int_1^\infty (f(x) - g(x))\, dx$$

$$= \int_1^\infty \left(\frac{1}{x} - \frac{4x}{1+4x^2}\right) dx.$$

$$= \left[\ln x - \frac{1}{2}\ln(1+4x^2)\right]_1^\infty$$

$$= \left[\ln \frac{x}{\sqrt{1+4x^2}}\right]_1^\infty$$

$$= \lim_{t\to\infty} \left[\ln \frac{x}{\sqrt{1+4x^2}}\right]_1^t$$

$$= \lim_{t\to\infty} \left(\ln \frac{t}{\sqrt{1+4t^2}} - \ln \frac{1}{\sqrt{5}}\right)$$

$$= \lim_{t\to\infty} \left(\ln \frac{1}{\sqrt{\frac{1}{t^2}+4}}\right) - \ln \frac{1}{\sqrt{5}}$$

$$= \ln \frac{1}{2} - \ln \frac{1}{\sqrt{5}} = \ln \frac{\sqrt{5}}{2}.$$

Answer: $\ln \frac{\sqrt{5}}{2}$.

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5B,

NO CALCULATOR ALLOWED

Work for problem 5(a)

$$g'(x) = \frac{4-16x^2}{(1+4x^2)^2}$$

when $g'(x) = 0$, $g(x)$ reach extremas

$$4 - 16x^2 = 0$$

$$x = \pm\frac{1}{2}$$

$g(\frac{1}{2}) = 1$ $\quad$ $g(-\frac{1}{2}) = -1$

since $g(\frac{1}{2}) > g(-\frac{1}{2})$,

$g(\frac{1}{2}) = 1$ is the absolute maximum

$g(-\frac{1}{2}) = -1$ is the absolute minimum

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Continue problem 5 on page 13.

Work for problem 5(b)

$f(x) = g(x)$

$\frac{1}{x} = \frac{4x}{1+4x^2}$

$4x^2 = 1+4x^2$

x does not exist

therefore there is no intersections between f(x) and g(x)

the area is equal to $\int_1^\infty f(x) - g(x)\, dx$

$\int_1^\infty f(x) - g(x)\, dx$

$= \int_1^\infty \frac{1}{x} - \frac{4x}{1+4x^2}\, dx$

$= \lim_{k\to\infty} \ln x - \frac{1}{2}\ln(1+4x^2) \Big|_1^k$

$=$

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Work for problem 5(a)

$g'(x) = \frac{8x\cdot 4x-(1+4x^2)\cdot 4}{(1+4x^2)^2} = \frac{32x^2-16x^2-4}{(1+4x^2)^2} = \frac{16x^2-4}{(1+4x^2)^2} = \frac{4(4x^2-1)}{(1+4x^2)^2}$

~~$\lim_{x\to 0} 16x^2-4 = -4$.~~

$\lim_{x\to 0} 1+4x^2 = 1$

$\therefore$ ~~$\lim_{x\to 0} g'$~~ when $g'(x)=0$ $x=\frac{1}{2}$

the maximum of $g(x)$ is $g(\frac{1}{2})=1$

~~$g'(x)$~~ $1+4x^2 \neq 0$

$\therefore$ the minimum doesn't exist.

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Continue problem 5 on page 13.

Work for problem 5(b)

the area $\cancel{=}$ $\int_1^\infty \left(\frac{1}{x} - \frac{4x}{1+4x^2}\right) dx$

$\int_1^\infty \frac{1}{x}\,dx + \int_1^\infty \frac{-4x}{1+4x^2}\,dx$ -

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (a) and 3 points in part (b). In part (a) the student finds $g'(x)$ and the critical point, but the analysis and conclusion did not earn any points. In part (b) the student sets up the correct improper integral and antidifferentiates correctly. Since the evaluation is not completed, the answer point was not earned.

Sample: 5C
Score: 4

The student earned 4 points: 3 points in part (a) and 1 point in part (b). In part (a) the student finds $g'(x)$ with a reversal of terms in the numerator, so 1 point was earned. The student finds the critical number and the maximum value and also asserts that there is no minimum. The third and fourth points were earned. In part (b) the student sets up the improper integral but does not do any additional work.

Question 6

The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$ on its interval of convergence.

(a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.

(b) Show that $y = f(x)$ is a solution to the differential equation $xy' - y = \frac{4x^2}{1+2x}$ for $|x| < R$, where R is the radius of convergence from part (a).

(a) $$\lim_{n\to\infty}\left|\frac{\frac{(2x)^{n+1}}{(n+1)-1}}{\frac{(2x)^n}{n-1}}\right| = \lim_{n\to\infty}\left|2x\cdot\frac{n-1}{n}\right| = \lim_{n\to\infty}\left|2x\cdot\frac{n-1}{n}\right| = |2x|$$

$|2x| < 1$ for $|x| < \frac{1}{2}$

Therefore the radius of convergence is $\frac{1}{2}$.

When $x = -\frac{1}{2}$, the series is $\sum_{n=2}^{\infty}\frac{(-1)^n(-1)^n}{n-1} = \sum_{n=2}^{\infty}\frac{1}{n-1}$.
This is the harmonic series, which diverges.

When $x = \frac{1}{2}$, the series is $\sum_{n=2}^{\infty}\frac{(-1)^n 1^n}{n-1} = \sum_{n=2}^{\infty}\frac{(-1)^n}{n-1}$.
This is the alternating harmonic series, which converges.

The interval of convergence for the Maclaurin series of f is $\left(-\frac{1}{2}, \frac{1}{2}\right]$.

$$5: \begin{cases} 1: \text{sets up ratio} \\ 1: \text{limit evaluation} \\ 1: \text{radius of convergence} \\ 1: \text{considers both endpoints} \\ 1: \text{analysis and interval of convergence} \end{cases}$$

(b) $$y = \frac{(2x)^2}{1} - \frac{(2x)^3}{2} + \frac{(2x)^4}{3} - \cdots + \frac{(-1)^n(2x)^n}{n-1} + \cdots$$
$$= 4x^2 - 4x^3 + \frac{16}{3}x^4 - \cdots + \frac{(-1)^n(2x)^n}{n-1} + \cdots$$
$$y' = 8x - 12x^2 + \frac{64}{3}x^3 - \cdots + \frac{(-1)^n n(2x)^{n-1}\cdot 2}{n-1} + \cdots$$
$$xy' = 8x^2 - 12x^3 + \frac{64}{3}x^4 - \cdots + \frac{(-1)^n n(2x)^n}{n-1} + \cdots$$
$$xy' - y = 4x^2 - 8x^3 + 16x^4 - \cdots + (-1)^n(2x)^n + \cdots$$
$$= 4x^2\left(1 - 2x + 4x^2 - \cdots + (-1)^n(2x)^{n-2} + \cdots\right)$$

The series $1 - 2x + 4x^2 - \cdots + (-1)^n(2x)^{n-2} + \cdots = \sum_{n=0}^{\infty}(-2x)^n$ is a geometric series that converges to $\frac{1}{1+2x}$ for $|x| < \frac{1}{2}$. Therefore $xy' - y = 4x^2\cdot\frac{1}{1+2x}$ for $|x| < \frac{1}{2}$.

$$4: \begin{cases} 1: \text{series for } y' \\ 1: \text{series for } xy' \\ 1: \text{series for } xy' - y \\ 1: \text{analysis with geometric series} \end{cases}$$

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$$

Interval of convergence?

Let's use the Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(2x)^{n+1}}{n}}{\frac{(-1)^n (2x)^n}{n-1}} \right| = \lim_{n\to\infty} |(-1)(2x)| = |2x| < 1$$

$|x| < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ → convergent

$x = \frac{1}{2}$ → $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n}{n-1}$

By Leibniz's criteria on convergence on series of alternative terms. $a_n > a_{n+1}$. $f(x)$ converges.

$x = -\frac{1}{2}$ $f(x) = \sum_{n=2}^{\infty} \frac{1}{n-1}$ → divergent.

$-\frac{1}{2} < x \leq \frac{1}{2}$ $\left(-\frac{1}{2}, \frac{1}{2}\right]$ is the interval of convergence for the Maclaurin series of f.

$R = \frac{1}{2}$

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Continue problem 6 on page 15

Work for problem 6(b)

$$xy' - y = \frac{4x^2}{1+2x}$$

$$f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$$

$$f'(x) = \sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{n-1} \cdot n \cdot x^{n-1}$$

$$xy' - y = \sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{n-1} \cdot n \cdot x^n - \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$$

$$= \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n (n-1)}{n-1} = \sum_{n=2}^{\infty} (-2x)^n$$

since $|x| < \frac{1}{2}$, $\sum_{n=2}^{\infty} (-2x)^n$ Converges (Ratio Test)

$\sum_{n=2}^{\infty} (-2x)^n$ is a geometric series $\left(1 + m + \cdots + m^n = \frac{1-m^{n+1}}{1-m}\right)$

$$\left(\lim_{n \to \infty} \sum_{k=0}^{n} m^k = \frac{1}{1-m}\right)$$

$$\therefore \sum_{n=2}^{\infty} (-2x)^n = \sum_{n=0}^{\infty} (-2x)^n - 1 - (-2x) = \frac{1}{1+2x} - 1 + 2x$$

$$= \frac{1 - 1 - 2x + 2x + 4x^2}{1+2x} = \frac{4x^2}{1+2x}$$

Therefore $y = f(x)$ is a solution to $xy' - y = \frac{4x^2}{1+2x}$ for $|x| < R$

Work for problem 6(a)

$$f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n(2x)^n}{n-1}$$

$$\lim_{n\to\infty}\left|\frac{\frac{\cancel{(-1)^{n+1}}(2x)^{\cancel{n+1}}}{n+1-1}}{\frac{\cancel{(-1)^n}\cancel{(2x)^n}}{n-1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(-1)(2x)(n-1)}{n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{n-1}{n}\cdot 2x\right|$$

$$=\lim_{n\to\infty}\left|\frac{1-\frac{1}{n}}{1}\cdot 2x\right|$$

$$=|2x|$$

~~For~~ By ratio test,

series is convergent when $|2x|<1$

$-\frac{1}{2}<x<\frac{1}{2}$

When $x=-\frac{1}{2}$

$$f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n(-1)^n}{n-1}=\sum_{n=2}^{\infty}\frac{1}{n-1}=\sum_{n=1}^{\infty}\frac{1}{n}$$ diverges (P-series)

When $x=\frac{1}{2}$

$$f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n(1)^n}{n-1}=\sum_{n=2}^{\infty}(-1)^n\frac{1}{n-1}$$ converges (alternative series)

Hence, the interval of convergence for $f(x)$ is $-\frac{1}{2}<x\leq\frac{1}{2}$

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Continue problem 6 on page 15.

Work for problem 6(b)

$$y = f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$$

$$y = (2x)^2 - \frac{(2x)^3}{2} + \frac{(2x)^4}{3} \dots \frac{(-1)^n (2x)^n}{n-1}$$

$$y' = 2\cdot 2(2x) - \frac{2\cdot 3(2x)^2}{2} + \frac{2\cdot 4(2x)^3}{3} \dots \frac{(-1)^n \cdot 2n \cdot (2x)^{n-1}}{n-1}$$

$$x y' = \frac{(-1)^n \cdot (2n)(2x)}{n-1}$$

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Work for problem 6(a)

$$\lim_{n\to\infty} \frac{(n+1)\text{th term}}{n\text{th term}} = \lim_{n\to\infty} \frac{\frac{(-1)^{n+1}(2x)^{n+1}}{n}}{\frac{(-1)^n (2x)^n}{n-1}}$$

$$= \lim_{n\to\infty} \frac{-2x(n-1)}{n} = 0$$

∴ The series must converge

The interval of convergence is all real number

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Continue problem 6 on page 15.

6C₂

NO CALCULATOR ALLOWED

Work for problem 6(b)

Putting $y = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$

$$y' = \sum_{n=2}^{\infty} \frac{2n(-1)^n (2x)^{n-1}}{n-1}$$

$$xy' = \sum_{n=2}^{\infty} \frac{n(-1)^n (2x)^n}{n-1}$$

$$xy' - y$$

$$= x\sum_{n=2}^{\infty} \frac{2n(-1)^n (2x)^{n-1}}{n-1} - \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$$

$$= \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1} [n-1]$$

$$= \sum_{n=2}^{\infty} (-1)^n (2x)^n$$

$$= \frac{(-1)^2 (2x)^2}{1+2x}$$

$$= \frac{4x^2}{1+2x} //$$

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Question 6

Sample: 6A
Score: 9

The student earned all 9 points. In part (a) an ideal solution would include an additional step at the beginning of the limit calculation. The student's presented work is correct.

Sample: 6B
Score: 6

The student earned 6 points: 5 points in part (a) and 1 point in part (b). In part (a) the student's work is correct. In part (b) the student finds the series for y', but what the student presents for xy' is not a series. Only the first point was earned.

Sample: 6C
Score: 4

The student earned 4 points: 1 point in part (a) and 3 points in part (b). In part (a) the student sets up the ratio test but does not evaluate the limit correctly. The first point was earned. In part (b) the student finds the series for y', xy', and $xy' - y$. The first 3 points were earned. The student has an algebraic error in the work leading to $\frac{4x^2}{1 + 2x}$, so the answer point was not earned.